2015

TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen. Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks 95

Section I

Total marks **(10)**

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided on the reverse side of this page.
- Allow about 15 minutes for this section

Section II

Total marks 85

- Attempt questions 11 – 16
- Answer on the blank paper provided, unless otherwise instructed
- Start a new page for each question
- All necessary working should be shown for every question
- Allow about 2 hour 45 minutes for this section

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet provided for Questions 1 – 10.

1. Evaluate $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$.

(A) 0
(B) undefined
(C) 4
(D) 1

2. Simplify : $\dfrac{\cos\left(\dfrac{\pi}{2} - \theta\right)}{\sin\left(\dfrac{\pi}{2} - \theta\right)}$

(A) 1
(B) $\cot\theta$
(C) $\tan\theta$
(D) $-\tan\theta$

3. The quadratic equation $x^2 - kx + 3 = 0$ has roots α and β.

What is the value of $\alpha^2\beta + \beta^2\alpha$?

(A) $k-3$
(B) $k+3$
(C) $-3k$
(D) $3k$

4. Which of the following is a primitive of the function $5 - \frac{1}{e^x}$?

(A) $5x + \frac{1}{e^x} + C$

(B) $5x - \frac{1}{e^x} + C$

(C) $5 - \frac{1}{e^{2x}} + C$

(D) $5x - \ln x + C$

5. Differentiate $-\frac{1}{2x^2}$

(A) $\frac{4}{x^3}$

(B) $\frac{1}{x^3}$

(C) $\frac{1}{x}$

(D) $-\frac{1}{x^3}$

6. The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point $(-2, 0)$. What are the values of m and b?

(A) $m = 9$ and $b = -18$
(B) $m = 9$ and $b = 18$
(C) $m = 12$ and $b = -18$
(D) $m = 12$ and $b = 18$

7. The table below shows the values of a function $f(x)$ for five values of x.

x	2	2.5	3	3.5	4
$f(x)$	4	1	−2	3	8

What value is an estimate for $\int_2^4 f(x)dx$ using Simpson's rule with these five values?

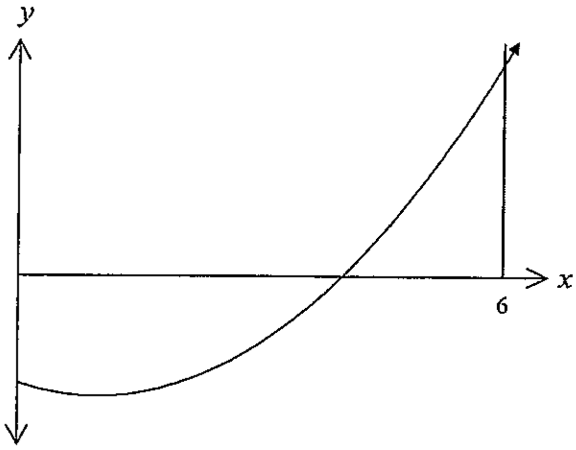
(A) 4
(B) 6
(C) 8
(D) 12

8. Ten kilograms of chlorine are placed in water and begin to dissolve. After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$. What is the value of k given that $A = 3.6$ and $t = 5$?

(A) −0.717
(B) −0.204
(C) 0.204

(D) 0.717

9. The diagram below shows the graph of $y = x^2 - 2x - 8$.



What is the correct expression for the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

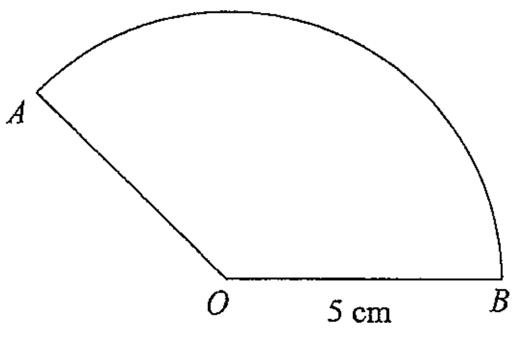
(A) $A = \int_0^5 x^2 - 2x - 8dx + \left|\int_5^6 x^2 - 2x - 8dx\right|$

(B) $A = \int_0^4 x^2 - 2x - 8dx + \left|\int_4^6 x^2 - 2x - 8dx\right|$

(C) $A = \left|\int_0^5 x^2 - 2x - 8dx\right| + \int_5^6 x^2 - 2x - 8dx$

(D) $A = \left|\int_0^4 x^2 - 2x - 8dx\right| + \int_4^6 x^2 - 2x - 8dx$

10. AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of 10π.



Not to scale

What is the arc length of the sector?

(A) 2π

(B) 4π

(C) 6π

(D) 10π

End of Section I

Section II

Total marks (60)
Attempt Questions 11-16
Allow about 1 hour 45 minutes for this section

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.

Question 11 (10 marks) Use a separate sheet of paper **Marks**

a) Factorise: $m^2 - n^2 + 5m - 5n$ **2**

b) Find in radians correct to three significant figures, the acute angle between the line $2x - 3y - 5 = 0$ and the positive x axis **2**

c) Simplify $\dfrac{4^{3n} \times 16^{1-3n}}{8^{-2n}}$ **2**

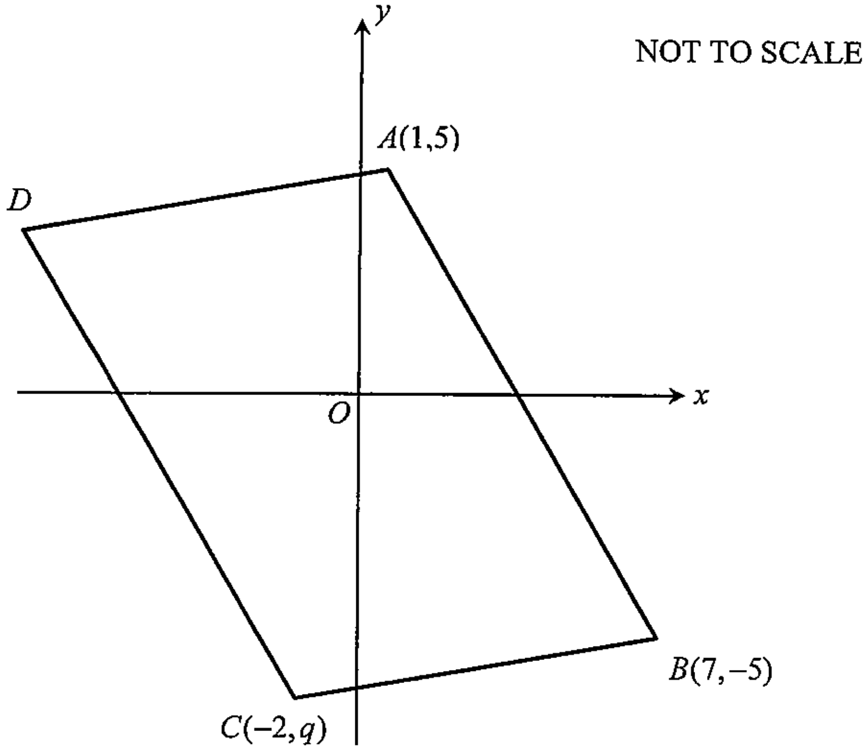
d) Differentiate $\dfrac{5}{\sqrt{2-3x^2}}$ **2**

e) Express $6\sqrt{5} - \dfrac{1}{\sqrt{5}-2}$ in the form $a + b\sqrt{5}$ where a and b are integers. **2**

End Question 11

Question 12 (15 marks) Use a separate sheet of paper

a) In the diagram below A, B, and C have coordinates $(1,5)$, $(7,-5)$ and $(-2,q)$ respectively. C is in the third quadrant and $ABCD$ is a parallelogram.



(i) Show that the equation of AB is $5x+3y-20=0$. **2**

(ii) Write down an expression, in terms of q, for the perpendicular distance from C to the line AB. **1**

(iii) Find the length of the interval AB. **1**

(iv) Given that the area of $ABCD$ is 100 square units, find the value of q. **3**

b) If the points $(-2a,\ 3)$, $(a-1,\ a-2)$ and $(a-3,\ a+1)$ are collinear, find the value of a. **2**

c) Differentiate $\sin(2x)+\cos\left(\dfrac{x}{2}\right)$ **2**

d) Find $\displaystyle\int \frac{x^3+x}{2x^2}\,dx$ **2**

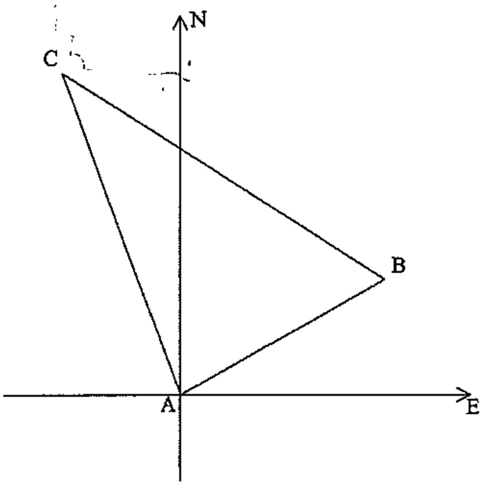
e) By completing the squares or otherwise find the vertex of the parabola

$$x^2-10x+15=2y$$ **2**

End Question 12

Question 13 (15 marks) Use a separate sheet of paper **Marks**

a) A ship sails 50 km from Port A to Port B on a bearing of 63°, then sails 130 km from Port B to Port C, on a bearing of 296°.



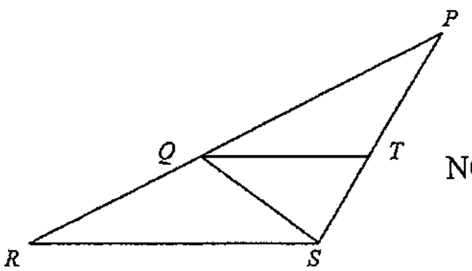
i) Copy the diagram and mark on it all the given information. **1**

ii) Show that $\angle ABC = 53^\circ$. **1**

iii) Find, to the nearest km, the distance of Port A from Port C. **1**

iv) Find the bearing of Port A from Port C, correct to the nearest degree. **2**

b) In the diagram, $QT \parallel RS$ and TQ bisects $\angle PQS$.



Copy the diagram into your answer booklet, showing this information.

i) Explain why $\angle TQS = \angle QSR$. **1**

ii) Prove that ΔQRS is isosceles. **2**

iii) Hence show that $PT : TS = PQ : QS$ **2**

Question 13 continued **Marks**

c) Solve for x : $\log_2(x+1)+\log_2(x+3)=3$ **3**

d) In a geometric sequence the third term is 18 and the seventh term is 1458.

Find the first term and the common ratio. **2**

End Question 13.

Question 14 (15 marks) Use a separate sheet of paper **Marks**

a) i) Show that $\dfrac{d}{dx}\{\log_e x\}^2 = \dfrac{2\log_e x}{x}$ **1**

ii) Hence evaluate $\displaystyle\int_1^e \frac{\log_e x}{x}\,dx$ **1**

b) A rain water tank which is full is drained so that at time 't' minutes, the volume of water V in litres is given by

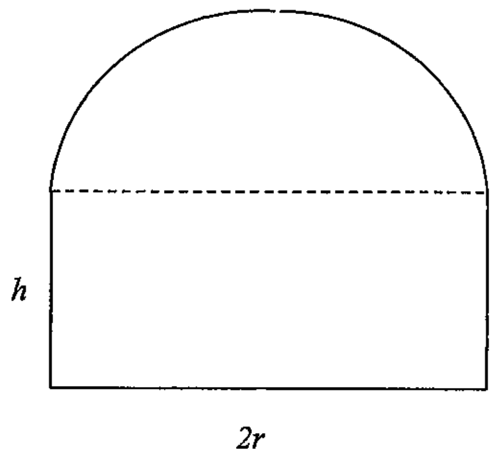
$V = 500\left(1-\dfrac{t}{60}\right)^2$ for $0 \le t \le 60$.

i) How much water was initially in the tank? **1**

ii) After how many minutes was the tank half full? **2**

iii) At what rate was the water draining when the time is 58 min. **2**

c) Lisa has designed a garden bed which consists of a rectangle and a semicircle as shown in the diagram. If the perimeter of the garden bed is to be 20 metres:



i. Find an expression for h in terms of r. **1**

ii. Show that the area of the garden bed can be given by the formula **1**

$$A = 20r - 2r^2 - \frac{1}{2}\pi r^2.$$

iii. Find the value of r that gives the maximum area and find this area. **3**

d) Consider the function $f(x) = \dfrac{x^2}{1+x^2}$. **3**

Show that the second derivative of this function is $f''(x) = \dfrac{2\left(1-3x^2\right)}{\left(1+x^2\right)^3}$

and hence find the values of x for which this function is concave up.

Question 15 (15 marks) Use a separate sheet of paper **Marks**

a) A quantity of radioactive material decays at a rate proportional to the amount, M, present at any time, t.

(i) Given that $M = M_0 e^{-kt}$ represents the mass of material in grams at any time t years after the material was produced, show that **1**

$$\frac{dM}{dt} = -kM$$

(ii) If initially there was 3500 g of material and after 4 years the mass had decayed to 2300 g, calculate k, correct to 4 significant figures. **2**

(iii) Determine the number of years needed for the material to decay to 25% of its original quantity. **2**

b) A particle moves in a straight line so that its velocity, v metres per second, at time t is given by $v = 3 - \dfrac{2}{1+t}$.

The particle is initially 1 metre to the right of the origin.

(i) Find an expression for the position x, of the particle at time t. **2**

(ii) Explain why the velocity of the particle is never 3 metres per second. **1**

(iii) Find the acceleration of the particle when $t = 2$ seconds. **2**

c) Consider the quadratic equation in x:

$$\left(p^2 + q^2\right)x^2 + 2q(p + r)x + \left(q^2 + r^2\right) = 0$$ **3**

Find an equation in terms of p, q *and* r such that the quadratic has real roots

d) Give a possible sketch for a curve which has the following defining features: **2**

When x = -5, 0 and 3,

$\dfrac{dy}{dx} = 0$.

Also $\dfrac{d^2y}{dx^2} > 0$ for x < -2½ and x > 1½, *while* $\dfrac{d^2y}{dx^2} < 0$ for - 2½ < x < 1½.

Question 16 (15 marks) Use a separate sheet of paper

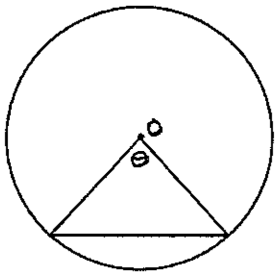
a) *TempIT* is an employment agency which specialises in contracting temporary employees. They have analysed the number of job applications received over the last five years. They found that the demand (D), measured in hundreds, for temporary employment at time (t years) is given by the function:

$$D(t) = 4\sin\left(\frac{\pi}{4}t\right) + 7$$

i) Find all the times in the next 12 years where demand will be at its peak **3**

ii) State the amplitude and period of $D(t)$ and sketch its graph for the first twelve years. **3**

b) The area of a circle radius r cm, is divided in the ratio 15:7 by a chord which subtends an angle of θ radians at the centre of the circle.



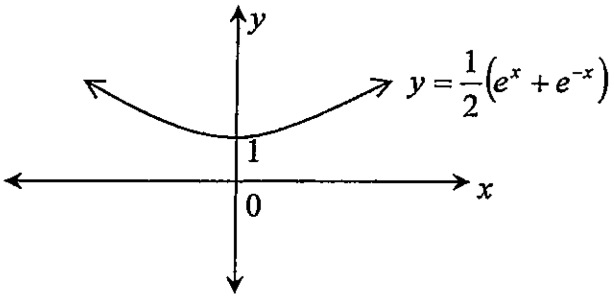
i) Show that the ratio of the areas of the major segment to minor segment is given by:

$$\frac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$$ **2**

ii) Given that $\pi = \dfrac{22}{7}$ prove that $\theta = 2 + \sin\theta$ **1**

Question 16 continues next page

c) The sketch of the catenary curve $y=\frac{1}{2}\left(e^{x}+e^{-x}\right)$ is given below. A catenary curve is the shape obtained when a chain or rope is strung between two points.

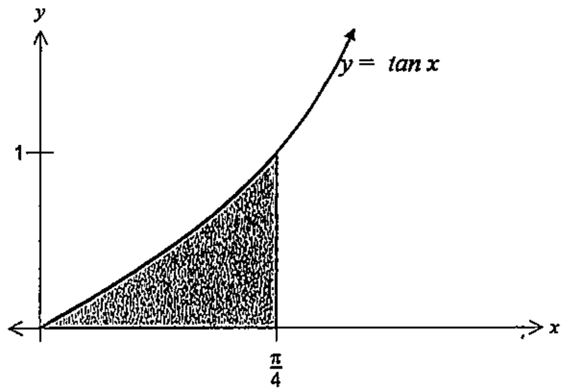


Calculate the volume of the solid generated when the curve **3**

$$y=\frac{1}{2}\left(e^{x}+e^{-x}\right)$$

is rotated about the x- axis between the ordinates $x =$ -3 and $x = 3$.

d) The area bounded by the curve $y = \tan x$, the lines $x = 0$ and $x = \frac{\pi}{4}$ and the x – axis is shaded below. **3**



Evaluate this area correct to 3 significant figures

End of Examination

Trial HSC– Mathematics - 2015

Section I – Multiple Choice Answer Sheet

Name_______________________________.

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

Start Here →

1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ○	C ○	D ●
4.	A ●	B ○	C ○	D ○
5.	A ○	B ●	C ○	D ○
6.	A ○	B ●	C ○	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ○	D ●
10.	A ○	B ●	C ○	D ○

1) $\lim \dfrac{(x-3)(x+1)}{x-3}$

$\lim\limits_{x \to 3} x+1 = 4$ (C)

2) $\dfrac{\cos(\frac{\pi}{2}-\theta)}{\sin(\frac{\pi}{2}-\theta)} = \dfrac{\sin\theta}{\cos\theta}$ (C)

$= \tan\theta$

3) $\alpha\beta(\alpha+\beta)$

$\frac{c}{a} \times \frac{-b}{a}$

$3 \times k = 3k$ (D)

4) $\int 5 - e^{-x}\,dx$

$= 5x + e^{-x} + c$ (A)

5) $\frac{d}{dx} \; -\frac{1}{2}x^{-2} = x^{-3}$ (B)

$= \frac{1}{x^3}$

6) $\frac{dy}{dx} = 3x^2 - 3$

at $x = -2$ $\quad m = 12 - 3$

$= 9$

$y - 0 = 9(x+2)$

$y = 9x + 18$

$m = 9 \quad b = 18$ (B)

7) $\frac{.5}{3}\left(4 + 8 + 4(1+3) + 2\times -2\right)$

$\frac{.5}{3}\left(12 + 16 - 4\right)$

4 (A)

8) $A = 10e^{-kt}$

$3.6 = 10e^{-5k}$

$\ln(.36) = -5k$

$k = 0.204$ (C)

9) $(x-4)(x+2) = 0$

cuts at $x = 4$ and -2

$\left|\int_0^4 \right| + \int_4^6$ (D)

10) $\frac{1}{2}r^2\theta = 10\pi$

$\frac{25}{2}\theta = 10\pi$

$\theta = \frac{4}{5}\pi$

$L = r\theta$

$= 5 \times \frac{4}{5}\pi$

$= 4\pi$ (B)

QUESTION 11

a) $(m-n)(m+n)+5(m-n)$

$=(m-n)(m+n+5)$

b) $\tan\theta = m$

$2x-5=3y$

$\frac{2}{3}x-\frac{5}{3}=y$

$m=\frac{2}{3}$

$\theta = 0.588.$

c) $\frac{2^{6n}\times 2^{4(1-3n)}}{(2^3)^{-2n}}$

$=\frac{2^{6n}\times 2^{4-12n}}{2^{-6n}}$

$=\frac{2^{4-6n}}{2^{-6n}}$

$=2^4$

d) $5(2-3x^2)^{-1/2}$

$\frac{dy}{dx}= \frac{-1}{2}\times 5(2-3x^2)^{-3/2}\times -6x$ Accept to here

$=\frac{15x}{\sqrt{(2-3x^2)^3}}$

e) $6\sqrt{5}-\frac{1}{\sqrt{5}-2} = 6\sqrt{5}-\frac{\sqrt{5}+2}{5-4}$

$=6\sqrt{5}-\sqrt{5}-2$

$=-2+5\sqrt{5}$

$a=-2$

$b=5$

QUESTION 12

a) i) $m = \dfrac{5 - -5}{1 - 7}$

$= \dfrac{10}{-6}$

$= -\dfrac{5}{3}$

$y - 5 = -\dfrac{5}{3}(x - 1)$

$3y - 15 = -5x + 5$

$5x + 3y - 20 = 0$

ii) $d = \dfrac{|5(-2) + 3 \cdot q - 20|}{\sqrt{5^2 + 3^2}}$

$= \dfrac{|3q - 30|}{\sqrt{34}}$

iii) $AB = \sqrt{6^2 + 10^2}$

$= \sqrt{136}$

$= 2\sqrt{34}$

iv) Area $= b \times h$

$2\sqrt{34} \times \dfrac{|3q - 30|}{\sqrt{34}} = 100$

$|3q - 30| = 50$

$\therefore\ 3q - 30 = 50$ or $3q - 30 = -50$

$3q = 80$ or $3q = -20$

$q = 26\tfrac{2}{3}$ $\qquad q = -\tfrac{20}{3}$

since q in 3rd quad $q = -\tfrac{20}{3}$

T.

b) $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{a-2-3}{a-1+2a} = \dfrac{a+1-3}{a-3+2a}$

$\therefore \dfrac{a-5}{3a-1} = \dfrac{a-2}{3a-3}$

$(3a-3)(a-5) = (a-2)(3a-1)$

$3a^2 - 18a + 15 = 3a^2 - 7a + 2$

$11a = 13$

$a = \dfrac{13}{11}$

c) $2\cos 2x - \frac{1}{2}\sin\left(\frac{x}{2}\right)$

d) $\int \dfrac{x^3}{2x^2} + \dfrac{x}{2x^2}\,dx$

$= \int \dfrac{x}{2} + \dfrac{1}{2x}\,dx$

$= \dfrac{x^2}{4} + \dfrac{1}{2}\ln x + c$

e) $x^2 - 10x = 2y - 15$

$x^2 - 10x + 25 = 2y - 15 + 25$

$(x-5)^2 = 2(y+5)$

vertex $(5, -5)$

QUESTION 13 5.

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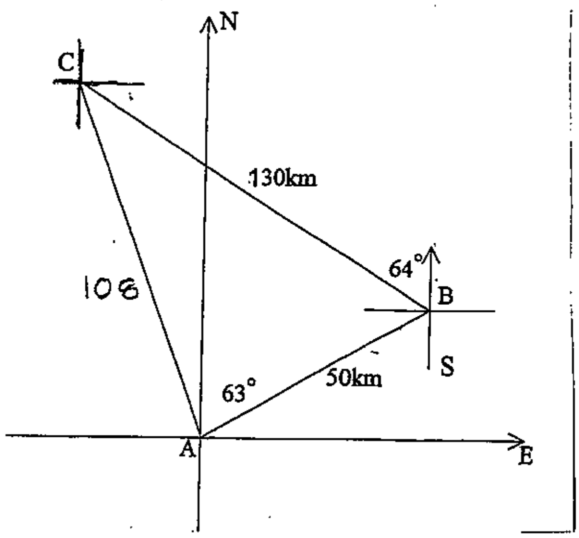


ii) $\angle ABS = 63°$ (alt $\angle$'s)

$\angle ABC = 296 - (180 + 63)$

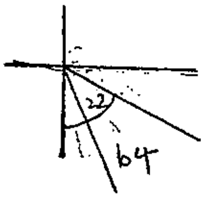
$= 53°$

iii) $AC^2 = 130^2 + 50^2 - 2 \times 130 \times 50 \times \cos 53$

$= 11567$

$AC = 108$ km

iv)



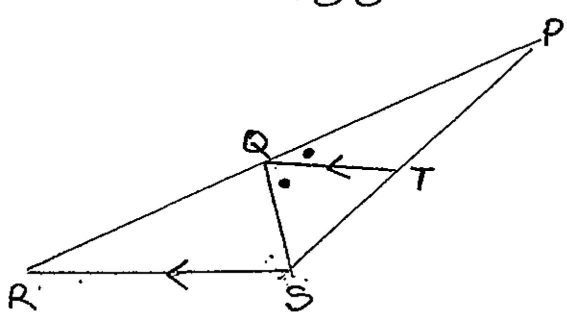
$\frac{\sin C}{50} = \frac{\sin 53}{108}$

$C = 22°$

$\therefore$ bearing is $180 + (64 - 22)$

$= 138°$

b)



i) $\angle TQS = \angle QSR$

equal alternate angles $QT \parallel RS$

ii)

ii) $\angle QRS = \angle PQT$ (corresponding $\angle$ equal $QT \parallel RS$)

and $\angle PQT = \angle TQS = \angle QSR$

$\therefore \angle QRS = \angle QSR$

$\therefore \triangle QRS$ isosceles (equal base $\angle$'s)

iii) Draw line through $P \parallel$ to QT.

$PT : TS = PQ : QR$ (ratio of intercepts on $\parallel$ lines)

but $QR = QS$

$\therefore PT : TS = PQ : QS$

c) $\log_2 (x+1)(x+3) = 3$

$\therefore (x+1)(x+3) = 2^3$

$x^2 + 4x + 3 = 8$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = 1$ as $x > 0$

d) $T_3 = ar^2$ $\quad$ $T_7 = ar^6$

$18 = ar^2$ $\quad$ $1458 = ar^6$

$$\frac{1458}{18} = \frac{ar^6}{ar^2}$$

$81 = r^4$

$r = \pm 3$

$ar^2 = 18$

$a \times 9 = 18$

$a = 2$

QUESTION 14

a) $\frac{d}{dx}\{\log_e x\}^2 = 2 \cdot \log_e x \cdot \frac{1}{x}$

$= \frac{2\log_e x}{x}$

ii) $\int_1^e \frac{\log_e x}{x} = \left[\frac{1}{2}(\log_e x)^2\right]_1^e$

$= \frac{1}{2}(\log_e e)^2 - \frac{1}{2}(\log_e 1)^2$

$= \frac{1}{2} \times (\log_e e)^2 - \frac{1}{2} \times 0$

$= \frac{1}{2}$

b) $t = 0$

i) $V = 500$

ii) $V = 250$ for half

$250 = 500\left(1 - \frac{t}{60}\right)^2$

$\frac{1}{2} = \left(1 - \frac{t}{60}\right)^2$

$\pm\sqrt{\frac{1}{2}} = 1 - \frac{t}{60}$

$\frac{t}{60} = 1 - \frac{1}{\sqrt{2}}$

$t = 0 \cdot 29 \times 60$ or $102 \cdot 42$ mins – reject $(60 - 30\sqrt{2})$

$= 17 \cdot 57$ min.

iii) $V = 500\left(1 - \frac{t}{60}\right)^2$

$\frac{dV}{dt} = 1000\left(1 - \frac{t}{60}\right) \times -\frac{1}{60}$

$= \frac{-1000}{60}\left(1 - \frac{t}{60}\right)$

$t = 58$ $\frac{dV}{dt} = \frac{-1000}{60}\left(1 - \frac{58}{60}\right)$

$= \frac{5}{9}$ Litres/min

c) $P = 2r + 2h + \frac{1}{2} \times 2\pi r$

$20 = 2r + 2h + \pi r$

$\frac{20 - 2r - \pi r}{2} = h$

or $10 - r - \frac{\pi r}{2} = h$

ii) $A = L \times b + \frac{1}{2}\pi r^2$

$= 2rh + \frac{1}{2}\pi r^2$

$= 2r\left(10 - r - \frac{\pi r}{2}\right) + \frac{1}{2}\pi r^2$

$= 20r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$

$= 20r - 2r^2 - \frac{1}{2}\pi r^2$

iii) $\frac{dA}{dr} = 20 - 4r - \pi r$

for a max $\frac{dA}{dr} = 0$

$4r + \pi r = 20$

$r = \frac{20}{4+\pi}$

$\doteqdot 2.8$

for a max

$\frac{d^2A}{dr^2} < 0$

here $\frac{d^2A}{dr^2} = -4 - \pi$

$= -7.14$

< 0

$\therefore$ max

max Area $= 20 \times 2.8 - 2(2.8)^2 - \frac{1}{2} \times \pi \times 2.8^2$

$\doteqdot 28\ m^2$

max $A = \frac{400}{4+\pi} - \frac{4+\pi}{2} \cdot \frac{400}{(4+\pi)^2}$

$= \frac{400}{4+\pi} - \frac{200}{4+\pi}$

$= \frac{200}{4+\pi}\ u^2$

) $f(x) = \frac{x^2}{1+x^2}$

$f'(x) = \frac{(1+x^2).2x - x^2.(2x)}{(1+x^2)^2}$ ✓

$= \frac{2x + 2x^3 - 2x^3}{(1+x^2)^2}$

$= \frac{2x}{(1+x^2)^2}$

$f''(x) = \frac{(1+x^2)^2.2 - 2x.2.2x(1+x^2)}{(1+x^2)^4}$

$= \frac{2(1+x^2)\left[(1+x^2) - 4x^2\right]}{(1+x^2)^4}$

$= \frac{2(1-3x^2)}{(1+x^2)^3}$ ✓

concave up when

$f''(x) > 0$

$2(1-3x^2) > 0$

$1 - 3x^2 > 0$

$3x^2 < 1$

$x^2 < \frac{1}{3}$

$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = -kM_0 e^{-kt}$

$= -kM$ (since $M = M_0 e^{-kt}$)

ii) $t=0$ $M_0 = 3500$

$2300 = 3500e^{-4k}$

$\frac{23}{35} = e^{-4k}$

$k = -\frac{1}{4}\ln\left(\frac{23}{35}\right)$

$= 0.10496$

$= 0.1050$ (4 sf)

iii) 25% of 3500 = 875 g

$875 = 3500e^{-0.1050t}$

$-0.1050t = \ln\left(\frac{1}{4}\right)$

$t = 13.2$

will decay during 14^{th} year.

b) $v = 3 - \frac{2}{1+t}$ $\quad t=0 \quad x=1$

$x = 3t - 2\ln(1+t) + c$

$1 = 0 - 2\ln 1 + c$

$\therefore c = 1$

$x = 3t - 2\ln(1+t) + 1$

ii) since $\frac{2}{1+t} > 0$

then $v = 3 - \frac{2}{1+t}$

$\neq 3$

iii) $v = 3 - 2(1+t)^{-1}$

$\frac{dv}{dt} = a = 2(1+t)^{-2}$

$= \frac{2}{(1+t)^2}$

$t=2$ $\quad a = 2/9$

c) real roots $\Delta \geq 0$

$b^2 - 4ac \geq 0$

$[2q(p+r)]^2 - 4.(p^2+q^2)(q^2+r^2) \geq 0$

$4q^2(p^2+r^2+2pr) - 4(p^2q^2 + p^2r^2 + q^4 + q^2r^2) \geq 0$

$\cancel{4q^2p^2} + \cancel{4q^2r^2} + 8prq^2 - \cancel{4p^2q^2} - 4p^2r^2 - 4q^4 - \cancel{4q^2r^2} \geq 0$

$8prq^2 - 4p^2r^2 - 4q^4 \geq 0$

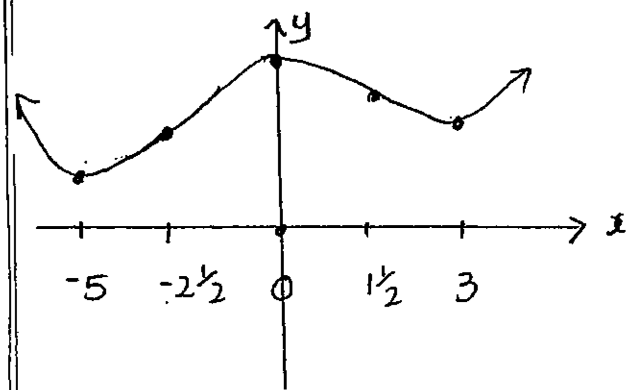
$q^4 - 2prq^2 + p^2r^2 \leq 0$

$(q^2 - pr)^2 \leq 0$

$\therefore$ as this is a square

$q^2 - pr = 0$

d)



QUESTION 16

a) $D(t) = 4\sin\left(\frac{\pi}{4}t\right) + 7$

find max when $D'(t) = 0$

$D'(t) = \pi\cos\left(\frac{\pi}{4}t\right)$

$0 = \pi\cos\left(\frac{\pi}{4}t\right)$

$\therefore \cos\left(\frac{\pi}{4}t\right) = 0$

when $\frac{\pi}{4}t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$t = 2, 6, 10$ years

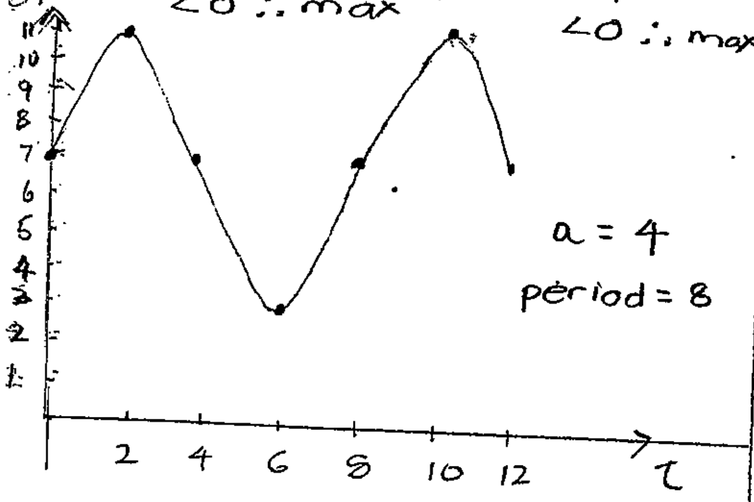
for max $D''(t) \leq 0$

$D''(t) = -\frac{\pi^2}{4}\sin\left(\frac{\pi}{4}t\right)$

$\therefore$ max at $t = 2$ and $t = 10$

$D''(t) = -\frac{\pi^2}{4} \times 1$ $<0 \therefore$ max

$-\frac{\pi^2}{4} \times 1$ $<0 \therefore$ max



$a = 4$

period $= 8$

) Area minor $= \frac{1}{2}r^2(\theta - \sin\theta)$

-ea major $=$

$\pi r^2 - \left(\frac{1}{2}r^2(\theta - \sin\theta)\right)$

$\therefore$ ratio $\dfrac{\pi r^2 - \frac{1}{2}r^2\theta + \frac{1}{2}r^2\sin\theta}{\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta}$

10

$= \dfrac{2\pi r^2 - r^2\theta + r^2\sin\theta}{r^2\theta - r^2\sin\theta}$

$\div r^2$

$= \dfrac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$

ii) $\frac{15}{7} = \dfrac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$

$15\theta - 15\sin\theta = 14\pi - 7\theta + 7\sin\theta$

$22\theta = 14\pi + 22\sin\theta$

if $\pi = \frac{22}{7}$

$22\theta = 14 \times \frac{22}{7} + 22\sin\theta$

$\theta = 2 + \sin\theta$

d) $\pi\int_{-3}^{3} y^2\,dx$ or $2\pi\int_{0}^{3} y^2\,dx$

$\therefore 2\pi\int_{0}^{3} \frac{1}{4}\left(e^{2x} + e^{-2x} + 2\right)dx$

$\frac{\pi}{2}\int_{0}^{3} e^{2x} + e^{-2x} + 2\,dx$

$\frac{\pi}{2}\left[\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x\right]_0^3$

$\frac{\pi}{2}\left[\frac{e^6}{2} - \frac{e^{-6}}{2} + 6 - \left(\frac{e^0}{2} - \frac{e^0}{2}\right)\right]$

$\frac{\pi}{2}\left[\frac{e^6}{2} - \frac{e^{-6}}{2} + 6\right]$

or $\frac{\pi}{2}\left[\frac{e^6 - e^{-6} + 12}{2}\right]$

d) $\int_0^{\frac{\pi}{4}} \tan x \, dx$

$= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} \, dx$

$= \left[-\ln(\cos x) \right]_0^{\frac{\pi}{4}}$

$= -\ln\left(\cos \frac{\pi}{4}\right) + \ln(\cos 0)$

$= -\ln \frac{1}{\sqrt{2}} + \ln 1$

$= 0.347$